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Identification of Systems Containing Nonlinear Element Using Constant Response Vibration Tests

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Abstract: Nonlinear behavior is important in the vibration test of engineering structures. In this study constant response vibration test is proposed for nonlinear element extraction. The method is based on the principle of harmonic balance method (HBM). The stiffness or damping can be regarded as constant for particular steady displacement or velocity response. The displacement or velocity is controlled as constant one in the test. Then the measured frequency response function (FRF) is obtained. The equivalent stiffness or damping is estimated by FRFs for particular vibration response level. The displacement-dependent stiffness and velocity-dependent damping are fitted to describe the unknown non-linearity. The nonlinear spring and damping force can be obtained by the fitting results combined with HBM with first-order expansion. Constant response vibration test is illustrated by experimental setup to verify its effectiveness. Experimental results show that the procedure is capable of achieving an accurate parameter identification of nonlinear damping and stiffness, which is hopeful for industrial application.

Keywords: non-linearity identification; sinusoidal sweep; modal test; constant response; frequency response function

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0 Introduction

Most practical assembled structures do not comply with an assumption of linearity. The complex non-linearity will be introduced into mechanical products due to the existing of nonlinear elements. The existing of nonlinear elements can no longer be neglected due to its amplitude-dependent properties[1-2]. Some research has been carried out in vibration analysis of nonlinear structures. Engineers need to identify the nonlinear elements in order to predict output vibration response during product design stage[3-4]. The investigation of nonlinear vibration is important for mechanical product improvement. As the dramatic improvement of vibration test technique, it is necessary to develop general method to extract nonlinear elements for industrial application.

Vibration test is necessary for nonlinear mathematical model update. Importantly, the presence of non-linearity can be detected in vibration test at larger excitation level[5-6]. The FRF might be distorted with increasing loading levels, the distortion of FRF for different excitation levels can offer meaningful insight into nonlinear behaviour. The variation of receptance also can be considered as evidence of structural non-linearity, and the jump phenomenon may even occur. As FRF in nonlinear system is not consistent, and it is dependent on response amplitude levels[7]. The evolution of modal parameters of nonlinear system for different output response often follows a particular law due to non-linearity, which is used to predict vibration response of nonlinear structures[8].

Some techniques had been proposed for nonlinear element identification. The Linearity Plot method is the most convenient method. Goege[9] provided a fast nonlinear characterization for large scale structures. The resonance frequencies and damping ratios of large aircraft were fitted as particular functions. Carrella[10]

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estimated the amplitude-dependent natural frequency and modal loss factor by modal test. The realization of non-linear identification is conducted by Restoring Force Surface (RFS) method. Feldman[11] utilized RFS to perform the nonlinear identification from measured vibration data. Yuan[12] compared RFS and Hilbert transform method based on vibration test of piezoelectric bimorph plate. The Describing Function method offers well description of nonlinear behaviour, which allows the quantification of practical non-linearity[13-15]. Wang[16] proposed the Equivalent Dynamic Stiffness Mapping technique, and the method was verified by classical numerical and experimental examples. Jalali[17] detected the multiple unknown elements in a nonlinear system by series FRFs and a reference linear one. Londoño[18] presented a technique for extraction of backbone curves of damped nonlinear systems. The technique for nonlinear stiffness identification needed the vibration response with steady-state.

There is need to extend the identification technique to extract structural non-linearity practically. In this study, constant response vibration test is developed to identify nonlinear stiffness and damping element in frequency-domain. The displacement amplitude is maintained in constant displacement vibration test. The velocity amplitude is maintained in constant velocity vibration test. The identification method depends on the basic idea that nonlinear structures exhibit linear behavior under steady response amplitude[19-20]. The idea is equivalent to the HBM with first-order expansion[21]. The displacement or velocity vary only within a limited range. Equivalent stiffness or damping is obtained for varied displacement or velocity response. The stiffness or damping function is constructed by fitting to describe the nonlinear elements. Response control vibration test is implemented to identify the pre-unknown nonlinear element of experimental setup. The stiffness and damping elements are approximated with ordinary polynomials. Finally, the predicted response is compared with the measured one, which shows a certain identification accuracy.

1 Estimation of Nonlinear Force

1.1 Theory

The basic theories are given in Refs. [20] and [21]. Some nonlinear structures exhibit linear behavior for steady response amplitude, thus equivalent linear analysis is applicable in response control vibration test.

The dynamic property of nonlinear system in steady-state response is corresponding to effective stiffness and damping. It is based on the equivalent linear concept. If the steady-state response can be considered to respond at the frequency of excitation. Only the primary harmonic is considered in describing function method, which is equivalent to application of HBM with first order expansion. The dynamic behaviour of a nonlinear system with amplitude-dependent damping and stiffness can be presented as Eq. (1).

$$m\ddot{x} + c(V)\dot{x} + k(X)x = f(t) \tag{1}$$

where m represents the modal mass, $k(X)$ is the stiffness function, $c(V)$ is the damping function, and V and X denote the displacement and velocity amplitude respectively. If the response amplitude is kept constant for the excitation frequency range, $k(X)$ and $c(V)$ in Eq. (1) are considered as constant.

If the nonlinear structure is excited by harmonic excitation and the force value is Eq. (2), where F is excitation amplitude, ω is the excitation frequency, and i denotes the imaginary unit.

$$f(t) = F e^{i\omega t} \tag{2}$$

The governing equation of the nonlinear system can be expressed as Eq. (3). The internal nonlinear force $f_n(x,\dot{x})$ is usually a function of displacement and velocity as Eq. (4).

$$m\ddot{x} + f_n(x,\dot{x}) = f(t) \tag{3}$$

$$f_n(x,\dot{x}) = f_k(x) + f_c(\dot{x}) \tag{4}$$

where x, $\dot{x}$ are the displacement and velocity, respectively.

This procedure will be illustrated through experimental setup with pre-unknown non-linearity. The identification using ordinary orthogonal polynomials is applied rather special function. The order of fitting polynomials is discussed. The nonlinear spring and damping force is assumed as follows:

$$f_k(x) = k_1 x + k_2 x|x| + k_3 x^3 + k_4 x^3|x| \tag{5}$$

$$f_c(\dot{x}) = c_1\dot{x} + c_2\dot{x}|\dot{x}| + c_3\dot{x}^3 + c_4\dot{x}^3|\dot{x}| \tag{6}$$

Moreover, the nonlinear elements of engineering structures is usually unknown before nonlinear characterization. The internal nonlinear force needs to be assumed and suitable nonlinear function needs to be selected for nonlinear modeling.

It is assumed that most of the vibration energy is concentrated in the excitation frequency[22-23]. The super-harmonics and the sub-harmonics are suppressed due to the damping effect. The steady-state vibration response only containing the primary harmonic component is presented as Eq. (7), where $\beta = \omega t + \varphi$ denotes the phase information of output response.

Substituting Eq. (7) into the polynomials nonlinear spring and damping force, it could be expanded in the Fourier series by Eq. (8).

$$x = X e^{i\beta} \tag{7}$$

$$f_n(x) = \frac{i}{\pi}\int_0^{2\pi} f_n(x,\dot{x})\, e^{-i\beta}\mathrm{d}\beta \tag{8}$$

When the Euler's formula is introduced,

$$e^{-i\beta} = \cos(\beta) - i\sin(\beta) \tag{9}$$

It yields:

$$f_n(x) = \frac{1}{\pi}\int_0^{2\pi} f_n(x,\dot{x})\sin\beta\mathrm{d}\beta + \frac{i}{\pi}\int_0^{2\pi} f_n(x,\dot{x})\cos\beta\mathrm{d}\beta \tag{10}$$

The stiffness and damping function is given to present equivalent stiffness and damping coefficient as

$$k(X) \approx \frac{1}{\pi X}\int_0^{2\pi} f_k(x)\sin\beta\mathrm{d}\beta \tag{11}$$

$$c(V) \approx \frac{1}{\pi\omega X}\int_0^{2\pi} f_c(\dot{x})\cos\beta\mathrm{d}\beta \tag{12}$$

$$k(X) = k_1 + \Delta_k \tag{13}$$

$$c(V) = c_1 + \Delta_c \tag{14}$$

For the polynomials nonlinear forcing, the stiffness and damping function of Eq.(5) and Eq.(6) take the form as Eq. (13) and Eq. (14), where

$$\Delta_k = \frac{8}{3\pi}k_2X + \frac{3}{4}k_3X^2 + \frac{32}{15\pi}k_4X^3$$

$$\Delta_c = \frac{8}{3\pi}c_2V + \frac{3}{4}c_3V^2 + \frac{32}{15\pi}c_4V^3$$

The stiffness and damping function is dependent of displacement and velocity response amplitude. It can be fitted by the equivalent stiffness and damping coefficient of response .

Then the nonlinear receptance expression of the system can be written in terms of stiffness and damping function as Eq. (15). The complex dynamic stiffness can be defined according to Eq. (15) as Eq. (16).The amplitude of nonlinear internal force in Eq. (16) can be presented as Eq. (17).

$$H_n = (-\omega^2 m + i\omega\cdot(c_1 + \Delta_c) + (k_1 + \Delta_k))^{-1} \tag{15}$$

$$D(\omega,X) = H_n^{-1} \tag{16}$$

$$N(\omega,X) = (H_n^{-1} + \omega^2 m)X \tag{17}$$

The equivalent stiffness and damping can be obtained by constant displacement/velocity vibration test. It is necessary and not strict to maintain a constant displacement or velocity response during test. The force level is adjusted to keep the constant output response within certain frequency range.

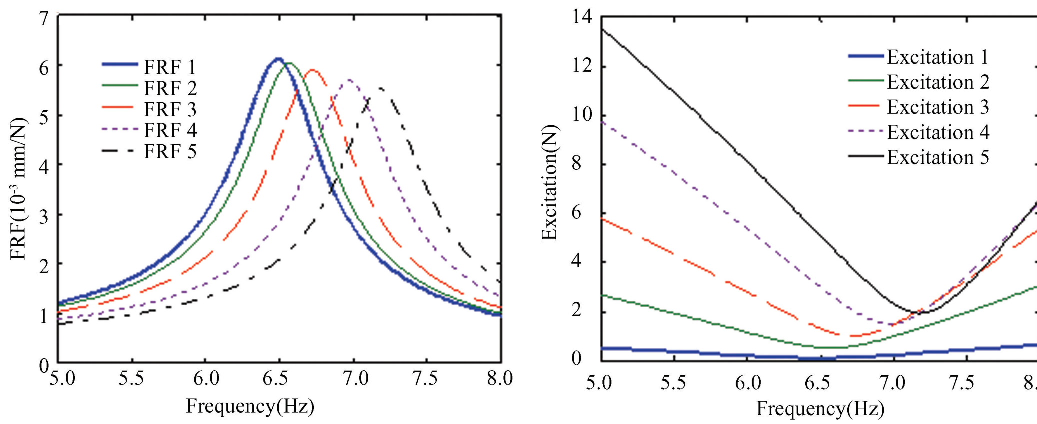
The constant response vibration test results in FRFs as Eq. (18), where $H_X(\omega,X)$ is FRF of displacement control test. $H_V(\omega,V)$ is FRF of velocity control test. $A(X)$ or $B(V)$ is the modal constant, $\omega_r(X)$ and $\omega_r(V)$ are the resonant frequency, and $\xi_r(X)$ and $\xi_r(V)$ are the damping coefficient. To be sure, the parameters vary with the control response. Note that Eq. (18) and Eq. (19) can be solved by an iterative approach. The real part of $H_X(\omega,X)$ is used to extract the resonant frequency $\omega_r(X)$. The imaginary part of $H_V(\omega,V)$ is used to extract damping coefficient $\xi_r(V)$. The resonant frequency and damping ratio vary with the response amplitude. The modal quantities need to be converted into the spatial parameter to obtain equivalent stiffness and damping.

$$H_X(\omega,X) = \frac{X(\omega)}{F(\omega)} = \frac{A(X)}{\omega_r^2(X) - \omega^2 + 2\mathrm{i}\,\xi_r(X)\omega\,\omega_r(X)} \tag{18}$$

$$H_V(\omega,V) = \frac{V(\omega)}{F(\omega)} = \frac{B(V)}{\omega_r^2(V) - \omega^2 + 2\mathrm{i}\,\xi_r(V)\omega\,\omega_r(V)} \tag{19}$$

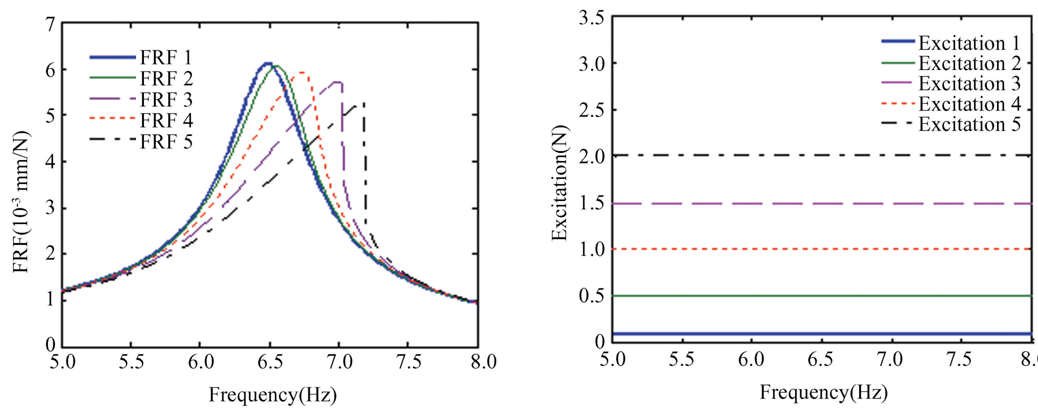
1.2 Identification Procedure

The nonlinear FRFs is important for identification method based on frequency domain. The constant response testing method is proposed. Fig. 1(a) shows the FRFs of response control vibration test. Fig. 1(b) shows the adjusted excitation level to maintain a constant response. During the procedure dedicated excitation signals are employed to control the response amplitude as constant using feedback control system. And the FRFs avoid nonlinear distortion. It exhibits non-deformed FRFs. The nonlinear structure are actually linearized at the fixed response level. The excitation force in response control vibration test is a variable. The conventional linear analysis tool is applicable for non-deformed FRFs. Fig. 2(a) shows the FRFs of excitation control vibration test. Fig. 2(b) shows the constant excitation level. As the increase of the excitation level, the non-linearity can be checked by the FRF deformation. In contrast to Fig. 1(a), the nonlinear FRF shifts due to inherent non-linearity. The distorted FRF can be used for the characterization of the non-linearity and judge its types. In contrast to Fig. 1(b), the excitation force in response control vibration test is changed in the frequency range.



(a) The obtained FRFs (b) The adjusted excitation

Fig. 1 FRFs by response control vibration test



(a) The obtained FRFs (b) The constant excitation

Fig. 2 FRFs by excitation control vibration test

The identification procedure is presented in Fig.3. First, response control vibration results in FRFs within certain frequency range. FRFs are used to extract modal data $\omega_r(X_i)$ and $\xi_r(V_i)$ by conventional linear analysis tools. obviously, it refers to a fixed value of the response amplitude. Then, the modal quantities is converted into spatial quantities. Equivalent stiffness is extracted by relationship $k_e = m\omega_r^2(X_i)$. Equivalent damping is extracted by relationship $c_e = 2\xi_r(V_i)m\omega_r(V_i)$. Thirdly, extraction tools is applied for each response control test. The discrete stiffness k_e is fitted as function of displacement amplitude. The discrete damping c_e is fitted as function of velocity amplitude. Stiffness function $k(X)$ is fitted by discrete stiffness k_e and dependent of X. Stiffness function $c(V)$ is fitted by discrete damping c_e and dependent of V. The fitting function reveal the inherent non-linearity information. The stiffness function and damping function are obtained to characterize the nonlinear element. The nonlinear elements are identified combining with stiffness and damping function and HBM with first-order expansion. In this procedure, it allows a accurate identification of non-linearity by stiffness function and damping function. This method has been successfully applied to identify nonlinear elements later.

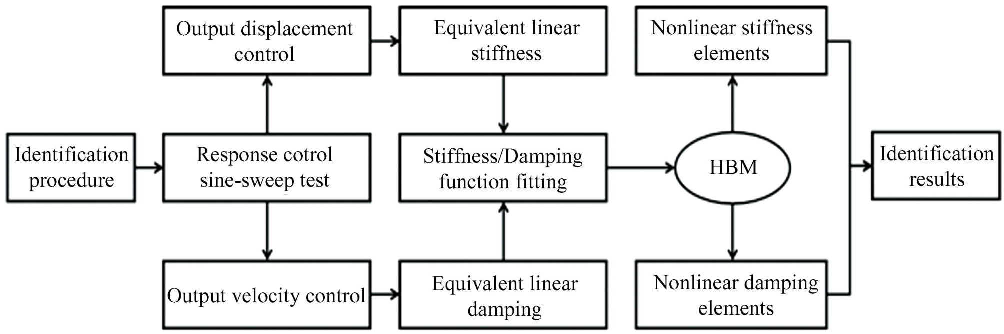


Fig. 3 Flowchart of nonlinear parameter identification by response control vibration test

2 Experimental Example

2.1 Experimental Setup

Structural joints are widely used in mechanical systems. Previous experiments have already shown that, due to the discontinuity of materials, frictional contact, mechanical micro-slippage or slapping along the contact interface, structural joints have become the main sources of structural non-linearity. The investigation with mechanical joint system is used in experimental setup. It produces non-linearity that could be varied by adjustment of pretension of mechanical joint. Note that the non-linearity is easy to be induced for low pretension level[24-26]. The torque wrench could regulate the pretension levels easily. Fig. 4 shows the real experiment rig for setup. A mass is resting on softening cushion, and four bolted joints are applied for connection. The mass is attached by mechanical joint, which introduces nonlinearity into system. All the measurements and excitation are measured by M+P data pickup. The mass is excited by shaker in horizontal direction through the stinger. Three accelerometers are placed at one side of the mass. The shaker's driving force is measured by one force sensor. And the nonlinear behavior of experimental setup is unknown. It presents non-linearity identification of experimental setup subsequently. Loosened connection is selected for subsequent constant response vibration test. It is used to validate the non-linear identification procedure.

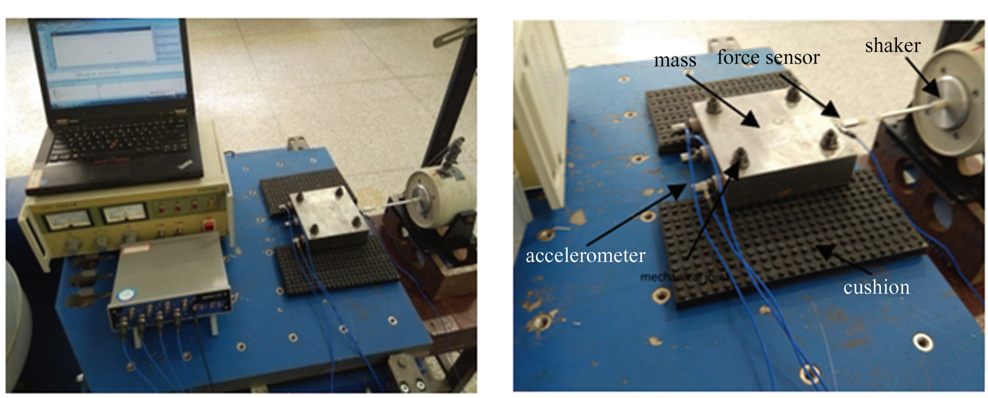
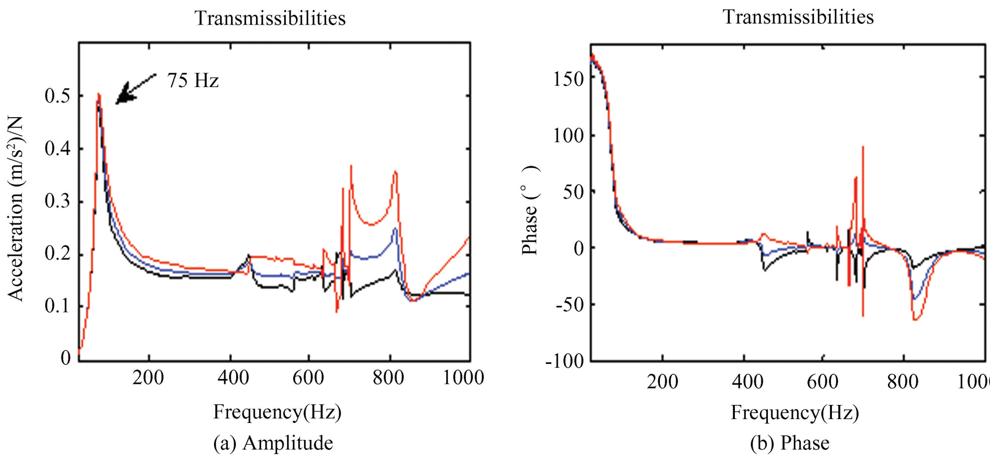


Fig. 4 Experimental setup representing configuration for vibration test

2.2 Normal Vibration Test

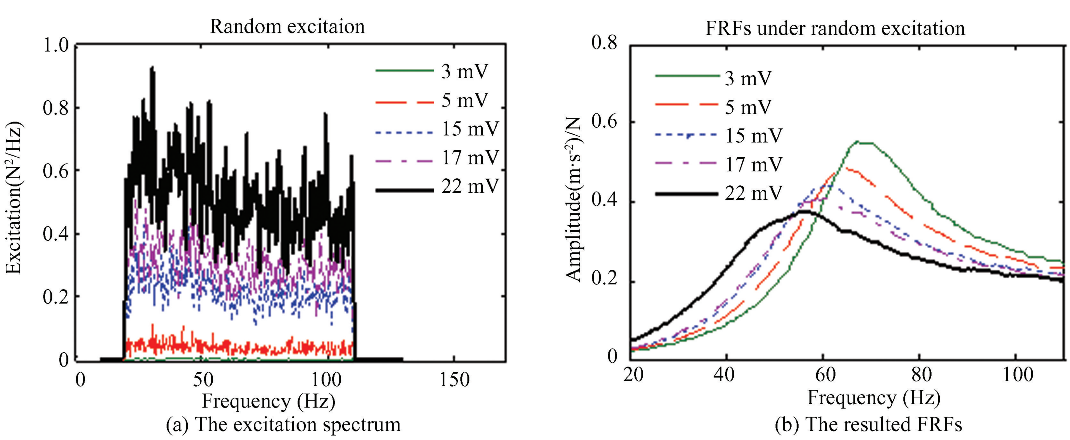
Before constant response vibration test, some normal vibration tests is conducted to detect vibration and nonlinear property of mechanical joint system. First, a hammer test is conducted to estimate the resonance frequency. The mass is excited by a hammer to judge the domaint resonance frequency. The input excitation and output response signals are measured in the horizontal direction. The sampling frequency is set to 2048 Hz with frequency resolution 0.125 Hz. The output responses of three accelerometers is seen in Fig.5. It is shown that the first resonance is around 75 Hz. The first mode is well separated from other modes. It can be considered as an isolated mode. The three accelerometers have nearly the same outputs for the first dominant resonance. It proves that the first mode shape is the dominant rigid motion in the horizontal direction.



(a) Amplitude (b) Phase

Fig. 5 FRFs are obtained from three accelerometers on the mass

Secondly, random vibration test is conducted to observe the structural non-linearity. The mass is excited by the shaker with the random vibration. A broad band random white noise is applied within the range of 20−110 Hz. The input excitation voltage and its spectrum range is seen in Fig. 6 (a). The FRFs under different input voltage levels around the resonance frequency in Fig. 6 (b). It shows that the FRFs are distorted and not overlay each other. The existence of non-linearity can be observed. Both resonance frequency and peak amplitude decreases as increase of excitation level. The frequency shift indicates the softening stiffness effect. The peak amplitude decreases indicates the hardening damping effect, which will be discussed later.



(a) The excitation spectrum (b) The resulted FRFs

Fig. 6 Set of FRFs for different input voltage levels using random excitation

At last, vibration test with one single frequency is conducted to verify whether the primary harmonic response is dominant. The mass is excited by a excitation with frequency of 60 Hz and amplitude of 7 N. The mass's acceleration response is shown in Fig 7. It shows that the sub- or super-harmonics also can be seen under excitation. And the primary harmonic response is dominant absolutely in comparison with sub- or super-harmonics. The steady-state response can be regarded containing only the primary harmonic component at the same frequency as the input force.

2.3 Constant Response Vibration Test

The response control vibration test is conducted for non-linearity identification. The most important mode is rigid motion in the horizontal direction. It reveals the nonlinear stiffness and damping of mechanical joints directly. Response control vibration is conducted by adjusting the shaker's output excitation. The shaker could regulate the excitation levels and frequency range as need easily. The three accelerometers have the same outputs for the first resonance frequency. The middle accelerometer placed on mass is selected as the response control point. The force sensor connected to the force bar is used to pick up the excitation signals. The output response is maintained as constant by feedback control system.

The closed feedback loop control is achieved by the M+P VibControl. The vibration data is analyzed by the M+P Data Pickup. The response control vibration test results in FRFs for modal information extraction. Different displacement and velocity response levels are selected for vibration response control.

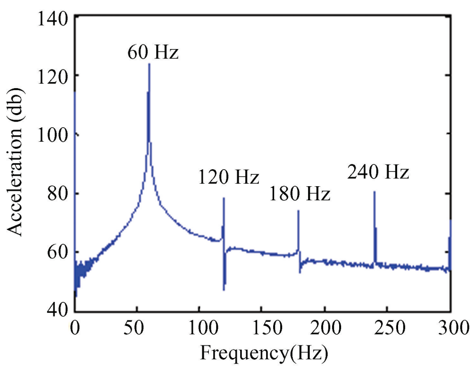
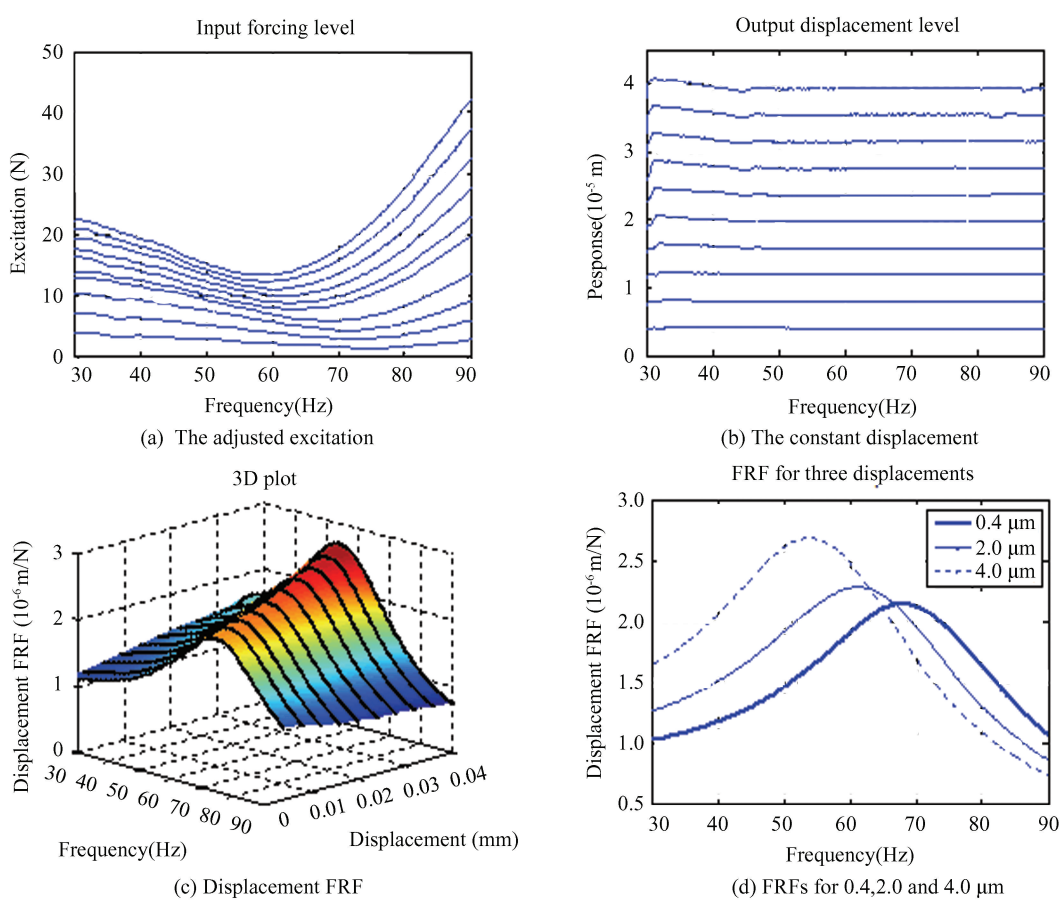


Fig. 7 Frequency domain responses of the acceleration (7 N, 60 Hz)

Constant displacement vibration test is conducted for equivalent stiffness identification and stiffness function fitting. Ten nearly constant output displacement levels from 4 μm to 40 μm is chosen and the experimental result confirms displacement control of mass is achieved successfully by closed-loop control. Fig.8(a) describes the adjusted excitation levels around resonance for maintain of constant displacement response. Fig.8(b) shows nearly constant displacement of mass in the horizontal direction. Fig. 8(c) shows FRFs ranging from 30 to 90 Hz in displacement response control vibration test. Fig. 8(d) shows FRFs of response control vibration test for 0.4 μm, 2 μm and 4 μm, individually.

Constant velocity vibration test is similar to constant displacement vibration test. Constant velocity vibration test is conducted for equivalent damping coefficient identification and damping function fitting. Ten sets of constant velocity tests are performed for the nonlinear modelling, and the velocity amplitude level ranges from 1 mm/s to 10 mm/s. Fig.9(a) describes the adjusted excitation levels around resonance for maintain of constant velocity response. Fig.9(b) shows nearly constant velocity of the mass. Fig.9(c) shows the FRFs for sets of velocity control vibration test. Fig.9(d) shows FRFs of response control vibration test for 1 mm/s, 5 mm/s and 10 mm/s, individually.



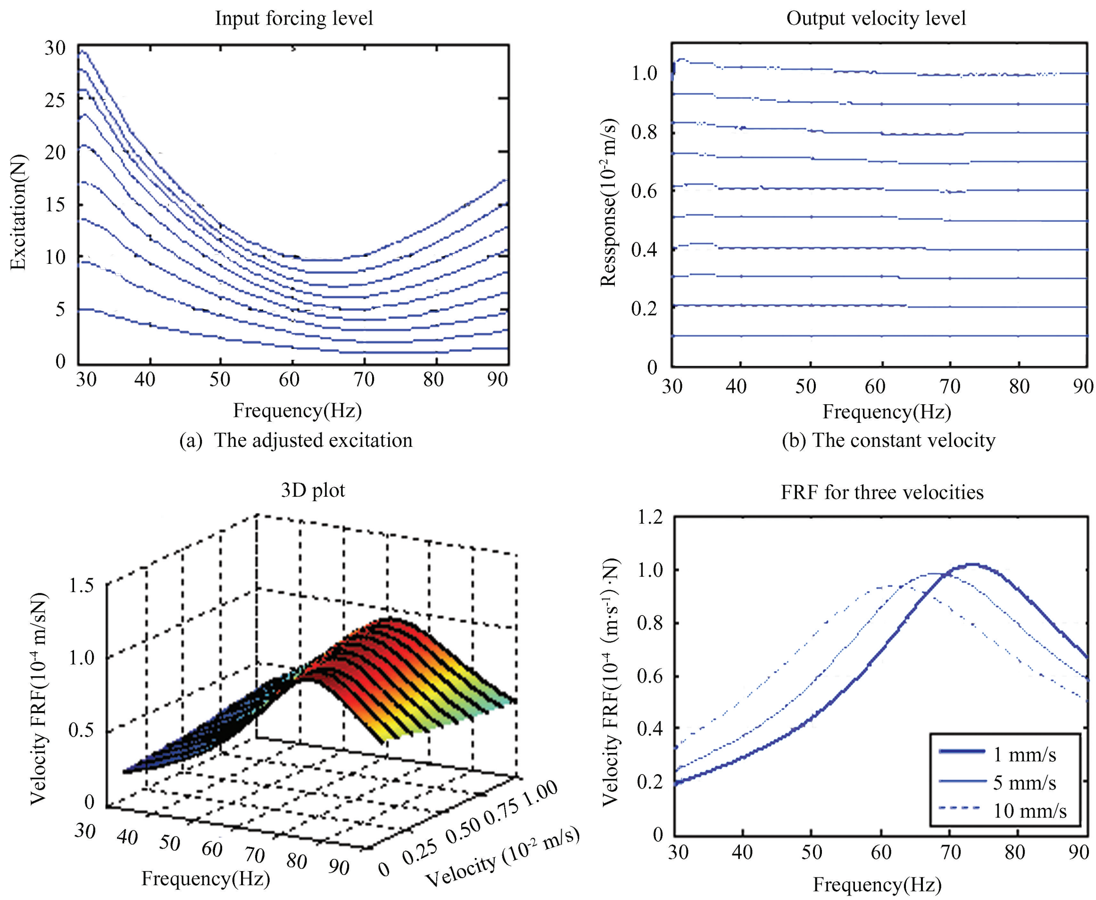
(a) The adjusted excitation

(b) The constant displacement

(c) Displacement FRF

(d) FRFs for 0.4,2.0 and 4.0 μm

Fig. 8 Results of constant displacement response vibration test



(a) The adjusted excitation (b) The constant velocity

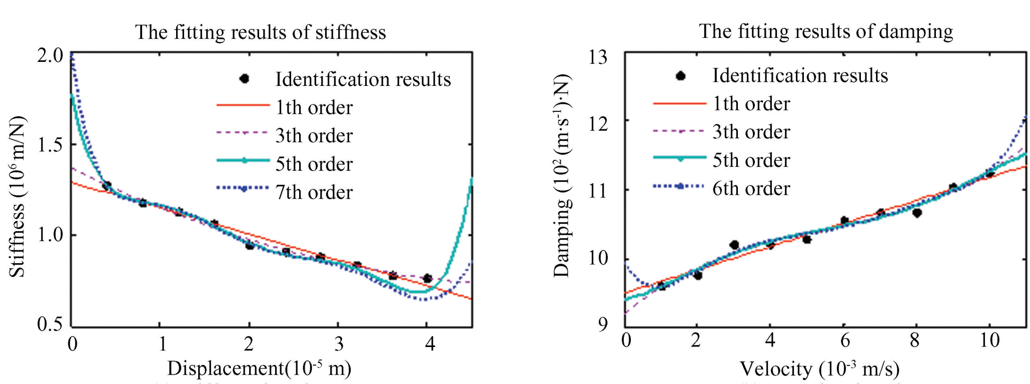
(c) Velocity FRF (d) FRFs for 15 and 10 mm/s

Fig.9 Results of constant velocity response vibration test

3 Modeling and Validation

3.1 Stiffness and Damping Function

The mass's displacement and velocity is controlled. The equivalent stiffness is related to displacement. The equivalent damping is related to velocity. Equivalent stiffness decreases by 34.3% with displacement. Equivalent damping coefficient increases by 16.6% with velocity. It confirms the nonlinearities of softening stiffness and hardening damping. The non-linearity of experimental setup is not pre-known. Its variation reveals the nonlinear property. The equivalent stiffness ranges from 4 μm to 40 μm is shown in Fig. 10(a). The equivalent damping ranges from 1 mm/s to 10 mm/s is shown in Fig.10(b).



(a) Stiffness function (b) Damping function

Fig.10 Fitting function obtained by vibration test

Series of equivalent stiffness and damping are fitted in ordinary polynomials. Combination of several basis functions (X , X^2 , V , V^2 , V^3) is sufficient to approximate $k(X)$ and $c(V)$ as:

$$k(X) = 1.366\,e^{6} - 2.817\,e^{10}X + 2.93\,e^{14}X^2 \quad (20)$$

$$c(V) = 919.8 + 5.121\,e^{4}V - 7.717\,e^{6}V^2 + 5.172\,e^{8}V^3 \quad (21)$$

Groups of various polynomial terms are selected as the basis functions for fitting seen in Fig. 8. It shows additional polynomial terms makes the numerical model more complex, which can not improve the goodness of fitting well.

3.2 Nonlinear Stiffness and Damping Force

The stiffness and damping function dependent of the response amplitude have been fitted. The physical parameter considering motion friction or connection stiffness is not established. Nonlinear spring and damping force with higher orders polynomial is effective. Substituting Eq. (18) and Eq. (19) into Eq. (1), it yields Eq. (22).

$$m\ddot{x} + (919.8 + 5.121\,e^{4}V - 7.717\,e^{6}V^2 + 5.172\,e^{8}V^3)\dot{x} + (1.366\,e^{6} - 2.817\,e^{10}X + 2.93\,e^{14}X^2)x = f(t) \quad (22)$$

Note that it is difficult to solve Eq. (21) by numerical method easliy. Numerical model is proposed to describe the nonlinear stiffness and damping force as Eq. (23).

$$m\ddot{x} + n_k(x) + n_c(\dot{x}) = f(t) \quad (23)$$

The stiffness and damping force are observed based on harmonic balance method and the fitting stiffness/damping function. The nonlinear spring and damping force are determined according to Eqs. (13) and (14) as Eqs. (24) and (25).

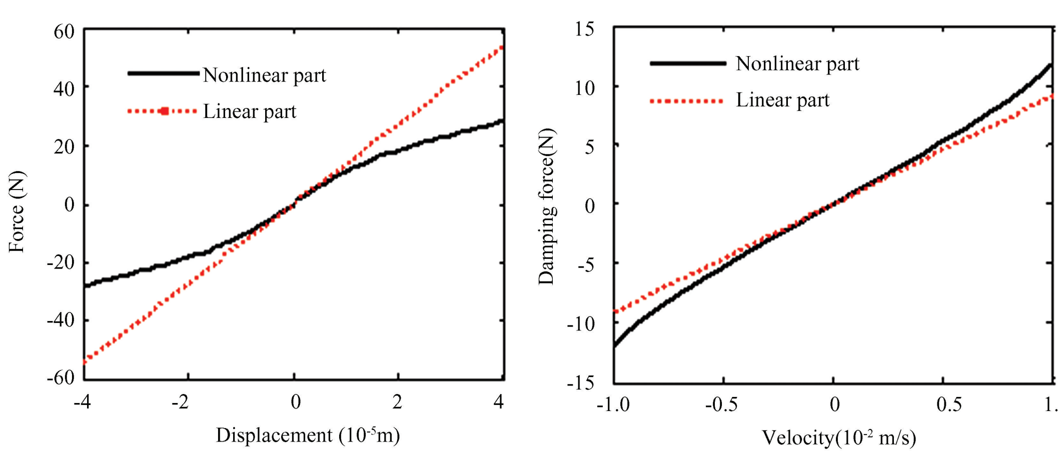
The function of the spring force ($x, x|x|, x^3$) presented as Eq. (24):

$$n_k(x) = 1.366\,e^{6}x - 3.319\,e^{10}x|x| + 3.907\,e^{14}x^3 \quad (24)$$

The function of the damping force ($\dot{x}, \dot{x}|\dot{x}|, \dot{x}^3, \dot{x}^3|\dot{x}|$) presented as Eq. (25):

$$n_c(\dot{x}) = 919.8\,\dot{x} + 6.033\,e^{4}\dot{x}|\dot{x}| - 10.289\,e^{6}\dot{x}^3 + 7.616\,e^{8}\dot{x}^3|\dot{x}| \quad (25)$$

Fig. 11 shows both linear and nonlinear part. It is related to the restoring force-displacement in Fig. 11(a) and the damping force-velocity relationship in Fig. 11(b). The nonlinear force reveals a typical softening stiffness. And the damping force increases with the velocity, which means a hardening nonlinear damping property. The estimating coefficient only describe the non-linearity within certain response range. The disadvantage is that the function has no physical meaning. The function is used to approximate restoring/damping force within a certain response range. For mechanical joint, the spring and damping force is approximated.



(a) Restoring force (b)Damping force

Fig .11 Linear and nonlinear part of restoring and damping force

3.3 Modelling Validation

The identified results are employed to validate the identification procedure. Eq. (23) is used to predict nonlinear vibration response of experimental setup. The measured response and the predicted acceleration response are shown in Fig. 12. The predicted and experimental responses show good agreements. It shows the nonlinear theoretical model has the ability to identify system with ideal accuracy. The evolution of natural frequency and response peak reveals the nonlinear type. It indicates the softening stiffness and hardening damping nolinearity. The drawback is that the identified results is effective within a limited response range. The prediction of a broader range of excitation levels is not so well by the identified nonlinear spring and damping force. The nonlinearity of experimental setup is not a particular function with certain characteristic. It needs a new fitting procedure to fit the stiffness and damping function for broader range of vibration response prediction.

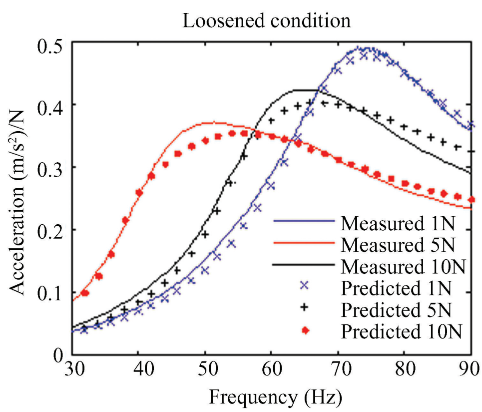


Fig.12 Measured and predicted responses for the loosened pretension level

4 Conclusions

The procedure has proved capable of nonlinear identification by response control vibration test. The output vibration response is maintained as constant by adjusting input excitation level. It is based that, at given response amplitude (displacement/velocity), the stiffness or damping can be considered as constant. And the equivalent stiffness and damping can be extracted from the measured FRFs by conventional linear analysis tools. The equivalent stiffness and damping is fitted as function of response amplitude. Stiffness function is dependent upon displacement and damping function is dependent upon velocity. The parameters or mathematical model with the unknown elements can be identified combined with HBM and the fitting function. Then the experiment is conducted. It represents the nonlinear force of experimental setup with suitable basis functions. The agreement between the predicted and measured results is satisfactory. It is linear with a very small forcing level and becomes nonlinear with increased excitation. So a well identification of nonlinearity of mechanical joints can be achieved within a certain and limited frequency and amplitude range.

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