Effect of crack face contact on dynamic stress intensity factors for a hole-edge crack

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Abstract: In order to determine the dynamic stress intensity factors (DSIFs) for a single edge crack at the center hole of a finite plate under a compressive step loading parallel to the crack, the finite element method was employed to solve the cracked plate problem. The square-root stress singularity around the crack tip was simulated by quarter point singular elements collapsed by 8-node two-dimensional isoparametric elements. The DSIFs with and without considering crack face contact situations were evaluated by using the displacement correlation technique, and the influence of contact interaction between crack surfaces on DSIFs was investigated. The numerical results show that if the contact interaction between crack surfaces is ignored, the negative mode I DSIFs may be obtained and a physically impossible interpenetration or overlap of the crack surfaces will occur. Thus the crack face contact has a significant influence on the mode I DSIFs.

Key words: crack; contact interaction; dynamic stress intensity factors (DSIFs); finite element method
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The components containing holes are widely used in engineering structures. A flaw or crack often exists at the hole-edge because of stress concentration, parts machining. It is necessary to determine stress intensity factors (SIFs) in order to evaluate the crack growth, residual strength and fatigue life of the cracked structure.

A number of theoretical, numerical and experimental studies for hole-edge cracks have been performed on evaluating the SIFs, but most of the investigations are concerned with the static SIFs. In fact, because of the effect of inertia, the SIFs of the hole-edge cracks under dynamic loading have great practical significance for the strength estimation of the cracked structure. However, the studies on the dynamic stress intensity factors (DSIFs) of the hole-edge cracks are relatively limited[1-4]. The influence of contact interaction between the crack surfaces on the results of numerical study of SIFs for a finite structure with hole-edge cracks under dynamic loading appears to be absent in the literature. When a cracked structure is subjected to dynamic loading, the influence of the loads is transferred to the cracks by means of stress waves through the material. During the interaction of the waves with crack, the edges of cracks may have a close contact and the forces of contact interaction arise in the contact regions. Analysis of the problem of static fracture mechanics demonstrates that the contact interaction between crack surfaces may significantly affect mechanics characteristics. In dynamic problems the effect can exceed those in the static case because the boundaries between contact and non-contact regions are variable in time and unknown beforehand[5-7]. Such boundaries make the contact problem non-linear and so far the only possible way to solve this problem is to use numerical methods such as the finite element method and boundary element method[8]. If the contact interaction of the crack surfaces is ignored, an application of a numerical approach will get the solution in which a negative mode I DSIF is obtained and the interpenetration or overlap of the crack faces is implied, which is physically impossible phenomena[5-11]. Therefore, a more accurate analysis for a cracked problem under dynamic loading has to include the contact interaction of the crack faces.

In the present paper, the DSIFs for a single edge crack emanating from the center hole in a finite plate subjected to impact compression stress parallel to the crack are analyzed using the general purpose finite element analysis software ANSYS, and the effects of contact interaction between the crack surfaces on dynamic fracture parameters are investigated.

1 Computational Method of DSIFs

In the fracture analysis, in order to simulate the square-root stress singularity around the crack tip, the quarter point singular elements collapsed by 8-node two-dimensional isoparametric elements are usually utilized. Quarter point, isoparametric, six-node, triangular,

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singular elements around the crack tip are depicted in Fig. 1, in which $AB = AE = L/4$ and L is the radial length or radius of first row of elements about the crack tip.

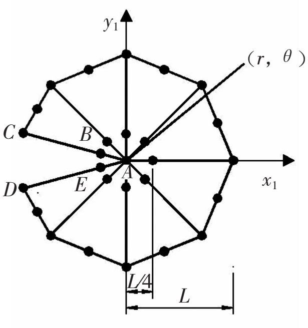

Fig. 1 Triangular quarter-node singular elements around the crack tip and local polar coordinate system

The DSIFs can be determined using the nodal displacement values of the quarter point singular elements around the crack tip in conjunction with the displacement correlation techniques. The asymptotic displacement field ahead of a crack tip in an isotropic linear elastic material can be written as[12]

$$u_{x1} = \frac{K_{\mathrm{I}}}{2\mu}\sqrt{\frac{r}{2\pi}}\cos\frac{\theta}{2}\left(\kappa - 1 + 2\sin^2\frac{\theta}{2}\right) + \frac{K_{\mathrm{II}}}{2\mu}\sqrt{\frac{r}{2\pi}}\sin\frac{\theta}{2}\left(\kappa + 1 + 2\cos^2\frac{\theta}{2}\right) \qquad (1)$$

$$u_{y1} = \frac{K_{\mathrm{I}}}{2\mu}\sqrt{\frac{r}{2\pi}}\sin\frac{\theta}{2}\left(\kappa + 1 - 2\cos^2\frac{\theta}{2}\right) - \frac{K_{\mathrm{II}}}{2\mu}\sqrt{\frac{r}{2\pi}}\cos\frac{\theta}{2}\left(\kappa - 1 - 2\sin^2\frac{\theta}{2}\right) \qquad (2)$$

where r and θ are coordinates in polar system with the crack tip at the origin, K_{I} and K_{II} are the stress intensity factors corresponding to modes I and II, ν is the Poisson's ratio and μ is the shear modulus, κ is $3 - 4\nu$ for plane-strain conditions, and u_{x1} and u_{y1} are displacement components in the x_1 and y_1 directions, respectively.

Evaluating Eqs. (1) and (2) at $\theta = \pm\pi$, the stress intensity factors for modes I and II can now be expressed as

$$K_{\mathrm{I}} = \lim_{r\to 0}\sqrt{2\pi}\,\frac{\mu}{1+\kappa}\,\frac{\Delta u_{y1}}{\sqrt{r}} \qquad (3)$$

$$K_{\mathrm{II}} = \lim_{r\to 0}\sqrt{2\pi}\,\frac{\mu}{1+\kappa}\,\frac{\Delta u_{x1}}{\sqrt{r}} \qquad (4)$$

where Δu_{x1} and Δu_{y1} are the relative displacement components of one crack surface with respect to the other at position r in the x_1 and y_1 directions, respectively.

SIFs depend on the values of $\Delta u/\sqrt{r}$ at position $r \to 0$, which need to be evaluated based on the nodal displacements and locations. With the linear extrapolation method[13], the value of $\Delta u/\sqrt{r}$ can be expressed as

$$\Delta u/\sqrt{r} = A_1 + A_2 r \qquad (5)$$

where A_1 and A_2 are constants.

The relative nodal displacements of the quarter-node singular elements on the crack surfaces (as shown in Fig. 1) can be evaluated according to the following equations

$$\Delta u_{x1}^{(1)} = u_{x1}^{B} - u_{x1}^{E},\ \Delta u_{x1}^{(2)} = u_{x1}^{C} - u_{x1}^{D} \qquad (6)$$

$$\Delta u_{y1}^{(1)} = u_{y1}^{B} - u_{y1}^{E},\ \Delta u_{y1}^{(2)} = u_{y1}^{C} - u_{y1}^{D} \qquad (7)$$

Using Eqs. (5), (6), (7), (3) and (4), and by adopting a linear extrapolation to the crack tip, the expressions for the DSIFs corresponding to modes I and II at arbitrary time t can be written as

$$K_{\mathrm{I}d}(t) = \sqrt{2\pi}\,\frac{\mu}{1+\kappa}\,\frac{[8\Delta u_{y1}^{(1)}(t) - \Delta u_{y1}^{(2)}(t)]}{3\sqrt{L}} \qquad (8)$$

$$K_{\mathrm{II}d}(t) = \sqrt{2\pi}\,\frac{\mu}{1+\kappa}\,\frac{[8\Delta u_{x1}^{(1)}(t) - \Delta u_{x1}^{(2)}(t)]}{3\sqrt{L}} \qquad (9)$$

The modes I and II DSIFs for stationary crack tip can now be calculated using the displacement correlation techniques provided that the displacement field of the cracked structure under any type of dynamic loading is solved.

2 Validation and Comparisons

In order to validate the finite element analysis techniques used in the present study, we provide some comparisons to the results given by Chen Y M[14]. The geometry and loading of the cracked plate problem considered by Chen Y M are depicted in Fig. 2. The material properties are as follows: shear modulus μ = 76.92 GPa, mass density ρ = 5000 kg/m^3 and the Poisson's ratio ν = 0.3. A state of plane strain is assumed. In validation analysis, we use the quarter point singular elements around crack tip and the 8-node regular elements in remaining part. The ratio of the length L of a quarter point singular element to the crack length $2a$ is taken as 0.01.

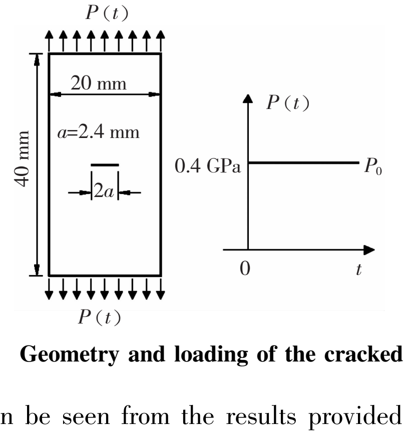

Fig. 2 Geometry and loading of the cracked plate

It can be seen from the results provided by Chen Y M in Fig. 3 that the mode I DSIFs in the range from 12 to 14 μs are less than zero, which means that the overlap of crack surfaces occurs at least in front of the crack tip. In order to examine the effect of crack closure on mode I situation, the contact areas between the crack surfaces are modeled by surface to surface contact elements. Here our analysis includes two contact cases, $kn = 0.0001$ for a lower normal contact stiffness and $kn = 1.0$ for a larger normal contact stiffness, where kn denotes the factor of normal contact stiffness in ANSYS. Only a half of the cracked plate is analyzed because of symmetry.

The comparison of the normalized DSIFs for mode I are shown in Fig. 3. The normalized mode I DSIF K_{I}^{n} is defined as

$$K_{\mathrm{I}}^{n}(t) = K_{\mathrm{I}}(t)/(p_0\sqrt{\pi a}) \tag{10}$$

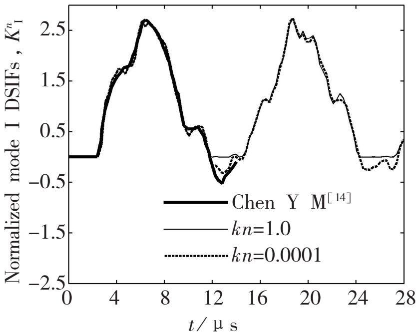

Fig. 3 Comparison of the normalized SIFs for mode I

As expected, when crack closure is allowed through a lower normal contact stiffness, the calculated results in the present study are in good agreement with the values given by Chen Y M. Hence, the finite element analysis techniques used are both efficient and reliable. It can be found from Fig. 3 that the results obtained from a larger normal contact stiffness and that calculated by Chen Y M have an excellent agreement in the range from 0 to 12 μs and show a divergence between 12 and 14 μs. These results indicate that the development of the negative mode I depends strongly on the assumption made for the crack surfaces, and the negative mode I and the overlap or interpenetration of the crack surfaces can be avoided by using a larger normal contact stiffness.

3 Model Illustration for a Hole-Edge Crack

The problem considered in the present study is a rectangular plate with a through-thickness crack at the center hole. Fig. 4 depicts the geometry and loading of the problem, in which the crack length a is equal to 10 mm. The linear elastic material properties are as follows: shear modulus $\mu = 80.769$ GPa, mass density $\rho = 7800$ kg/m^3 and the Poisson's ratio $\nu = 0.3$. The impact compression stress is $\sigma_0 = 1.0$ MPa. The problem is analyzed in linear elastic 2-D plane-strain conditions.

The finite element model is created using the general-purpose finite element analysis software ANSYS 5.7. The problem is analyzed with and without contact elements along the crack surfaces in order to examine the effect of crack closure on DSIFs. The finite element model, as shown in Fig. 5, contains 24 quarter-node singular elements around the crack tip and 6236 linear elastic isoparametric 8-node elements in remaining part. The ratio of the length L of a quarter point singular element to the crack length a is taken as 0.01. The contact areas along the crack surfaces are modeled by 36 surface to surface contact elements when contact interaction is considered. Total number of nodes in the model is 19206 hence the total number of degrees of freedom is 38412. The total computational time is 100 μs, which is divided into 600 time steps.

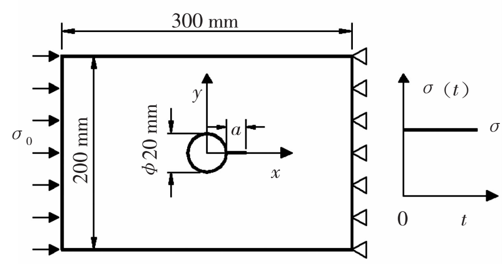

Fig. 4 Geometry and loading of the cracked structure

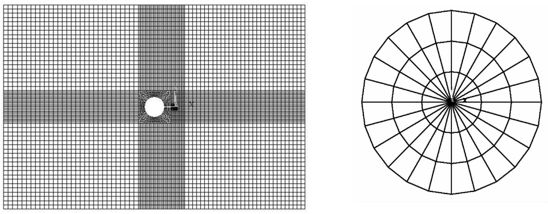
(a) Finite element model (b) Close-up view

Fig. 5 The elements around the crack tip

4 Results and Discussion

4.1 The Contact Pressure on the Crack Surfaces

The computed contact pressures versus time for three locations on the crack surfaces are shown in Fig. 6, in which curves P_L, P_M and P_{CT} are the normal contact pressures normalized by the step stress σ_0 at locations $r = a$, $r = a/2$ and at the crack tip, respectively. It can be seen from Fig . 6 that these three curves are

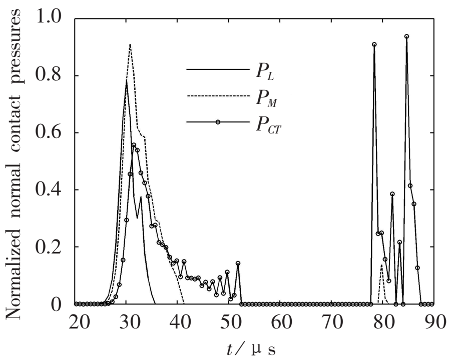

Fig. 6 Contact pressures in time on the crack surfaces

similar while the differences in the contact times and pressures at different locations are obvious. The existence of the contact pressures on the crack edges confirms that the edges of crack do have a close contact and the forces of contact interaction arise in the contact regions. So the interpenetration or overlap between crack surfaces will occur in dynamic analysis if the contact interaction of the crack surfaces is ignored.

4.2 The Dynamic Stress Intensity Factors

The calculated DSIFs versus time are normalized into non-dimensional that according to the following equation.

$$K_{\mathrm{I}d}^{n} = K_{\mathrm{I}d}(t)/K_{\mathrm{I}}^{s} \qquad (11)$$

$$K_{\mathrm{II}d}^{n} = K_{\mathrm{II}d}(t)/K_{\mathrm{I}}^{s} \qquad (12)$$

where $K_{\mathrm{I}d}(t)$ and $K_{\mathrm{II}d}(t)$ are given in Eqs. (8) and (9) and K_{I}^{s} is the static stress intensity factor corresponding to mode I under static compression loading σ_0.

The normalized DSIFs with and without contact elements along the crack surfaces are shown in Fig. 7, in which curves 2 and 1 correspond to the modes I and II DSIFs without contact interaction, curves 4 and 3 denote the modes I and II DSIFs with frictionless contact interaction.

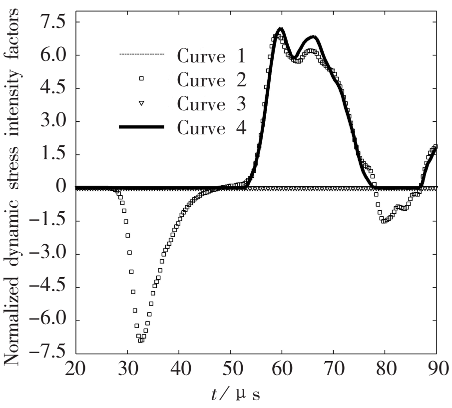

Fig. 7 The normalized DSIFs with and without contact elements along the crack surfaces

It can be seen from Fig. 7 that the mode II DSIFs are approximately zero owing to symmetry whereas the relatively larger differences in the mode I DSIFs without and with the contact interaction of the crack surfaces are found. The mode I DSIFs without contact interaction at crack edges, as shown in curve 2, present the larger negative values in the range from 28 to 46 μs, which imply that there is at least the serious interpenetration in the neighborhood of the crack tip. Curve 4 in Fig. 7 shows that when the contact interaction at the edges is considered, the larger negative mode I DSIFs in the range mentioned above are prevented and the maximum value of the mode I DSIFs in the range from 60 to 70 μs exceeds the corresponding value without contact interaction by 12%. These results indicate that the contact interaction between crack surfaces has significant influence on mode I DSIFs.

5 Conclusions

1) If the contact interaction between the crack surfaces is ignored, the negative mode I DSIFs may be obtained and a physically impossible interpenetration or overlap of the crack surfaces will occur.

2) The contact interaction between crack surfaces has significant influence on mode I DSIFs. The negative mode I and a interpenetration or overlap between the crack surfaces may be prevented by taking account of the contact interaction of the crack edges.

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